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Diffusion, Diffraction and Reflection in Semiconductor o.b. Devices

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Diffusion, diffraction and reflection in semiconductor o.b. devices

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For reasons of speed and economy, applications of optical bistability are likely to concentrate on solid-state devices of small volume. In this paper we discuss the modelling and optimization of such devices.

We report calculations of surface reflectivity optimization in the presence of absorption, and show that optical bistability in reflection offers significant advantages over transmission.

Small transverse dimensions, i.e. dense packing, are limited by cross-talk between neighbouring devices. We show that diffraction and diffusion give rise to qualitatively similar effects.

1. Introduction

The macroscopic problem in optical bistability is rather simple, at least at face value. Briefly, given a material with a known behaviour of nonlinear refractive index, one wishes to find the combination of material absorption and end-face reflectivities that will optimize the bistable response for a given distribution of input intensities. This apparently simple requirement hides a formidable mathematical and computational problem that has been tackled in certain cases, but for which no all-embracing model exists. This paper deals with certain aspects of the three phemonema mentioned in the title, but in reverse order. We first argue that the reflected, rather than the transmitted, beam is the more useful output from a bistable device. We then discuss the nature of the 'transverse effects' that arise when physical beams of finite transverse dimensions are considered. We report some recent computer results for this problem for the case of a local nonlinear response. Lastly, we consider the transverse effects of diffusion, when the induced free carriers responsible for the nonlinear refractive index in semiconductors such as indium antimonide are sufficiently long-lived and mobile to migrate significant distances in response to concentration gradients, thereby broadening and smoothing the induced refractive index distribution.

2. Reflection

In this section an analysis of bistable Fabry–Perot action in reflection is outlined. Full details are published elsewhere (Wherrett 1984). Reasons are given why reflection-mode may be preferable to transmission-mode operation in device applications.

Here, and below, our analysis will be framed in terms of a Fabry–Perot cavity with an intensity dependent refractive index, though many of the ideas will be relevant to a wider class of bistable devices. Given a suitable material and laser frequency, the parameters at the designer's disposal are the front (input) and back (transmission) face reflectivities, R_f and R_b (which can be varied by use of coatings), and the material absorption, αD, assumed linear (which can be varied by varying the thickness D of nonlinear material). For any given case, we can define an 'effective reflectivity',

$$R_\alpha = (R_f R_b)^{\frac{1}{2}} \exp(-\alpha D), \tag{1}$$

and a 'finesse factor',

$$F = 4R_\alpha/(1-R_\alpha)^2.$$

We also define a semi-round-trip phase shift, ϕ, defined as (modulo π)

$$\phi = (3\pi D/\lambda)\, n_2 \bar{I} - \delta, \tag{2}$$

where δ is the low-intensity cavity detuning, $\bar{I}$ the mean internal intensity, and the refractive index depends on intensity as

$$n(I) = n_0 + n_2 I.$$

It is then possible (Miller 1981; Wherrett 1984) to decouple the microscopic and macroscopic aspects of the bistability problem by scaling intensities in terms of the characteristic intensity of the material, $\lambda\alpha/(3\pi n_2)$. It then turns out, for example, that I_c, the critical intensity to obtain bistability, depends only a universal function of R_f, R_b and $e^{-\alpha D}$. I_c diverges if either R_f or R_b is zero, because there is then no feedback. Equally, as R_f tends to unity I_c diverges as it becomes impossible to get light into the cavity. In contrast, as R_b tends to unity I_c *decreases*, because $\bar{I}$ is enhanced owing to the lower back-face loss. For a 100% reflective back surface there is, of course no transmission whatever and I_c is minimized. One is thus led to examine the properties of the cavity reflectivity, R, given by

$$R = 1 - \frac{1}{(1+F\sin^2\phi)} \frac{(1+R_\alpha^2 - R_f - R_b\, e^{-2\alpha D})}{(1-R_\alpha)^2}. \tag{3}$$

The intensity dependence of R arises entirely from the $\sin\phi$ term: R is thus a minimum when $\sin\phi$ is zero (cavity on resonance) and vice versa, and shows hysteresis if T does. This means that the 'logical value' of R is complementary to that of the transmission, T, which is maximal on resonance. Of more immediate interest is the fact that the range of R between 'off' and 'on' can be large even if αD is substantial, as is shown in figure 1. This could be particularly useful in cascading optical logic devices where, perhaps because of material problems or fabrication problems, it is necessary to cope with substantial absorption. The figure shows clearly that in such a case transmission drops rapidly with αD but that a usable switching range is obtainable in reflection even for $\alpha D = 1$.

These considerations are elaborated elsewhere (Wherrett 1984); the main advantages of the reflection mode (with $R_b = 1$) are identified as (i) lower critical intensity, (ii) better absorption tolerance; (iii) less critical fabrication specifications for R_f, R_b and δ; and (iv) simpler cooling, because the entire back surface can be a heat sink. It should also be noted that the reflection signal is there anyway in Fabry–Perot devices, and would have to be suppressed in cascaded transmission-mode devices. Finally, Wherrett (1984) also demonstrates that optical logic devices and oscillators can be based on reflection-mode as well as on transmission-mode bistable devices.

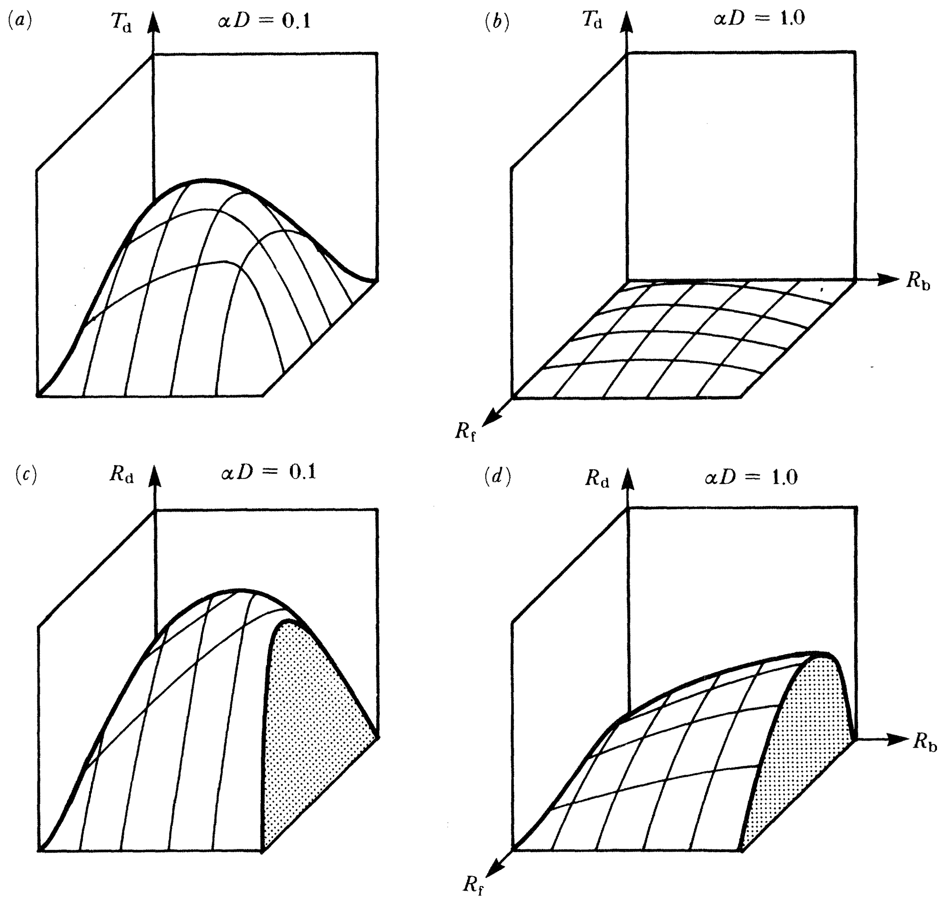

FIGURE 1. Differences between 'on' and 'off' signals: (*a*, *b*) the transmission difference, T_d, for $\alpha D = 0.1$ and 1.0; (*c*, *d*) the reflection difference, R_d, for $\alpha D = 0.1$ and 1.0.

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3. Diffraction

The above analysis is strictly valid only in the plane-wave limit, as indeed is most work in the theory of optical bistability. Consideration of the finite size and non-uniform intensity distribution of the input beam considerably complicates the analysis by introducing a number of 'transverse effects' associated with the intensity and refractive index gradients within the cavity.

Any finite intensity distribution clearly gives rise to a *diffractive* spreading (and phase shifting) of the beams. Because of the complex interplay between amplitude and phase distributions, diffraction is hard to analyse, even numerically, in nonlinear problems. Within the cavity the diffraction properties can be characterized by the *Rayleigh range* $\pi w^2/\lambda$, where w is the beam-width (for a Gaussian beam, the field amplitude varies as e^{-r^2/w^2}), and $\lambda (= \lambda_0/n_0)$ the wave-length in the medium. The Rayleigh range, z_R, is a measure of the distance over which a light beam remains collimated, and if it is small compared with the cavity length, L, there is strong diffractive coupling between different parts of the beam, and we may expect the beam to switch, if at all, as a unit (whole-beam switching (Moloney & Gibbs 1982)).

The power output will then show hysteresis qualitatively similar to that in plane-wave models. On the other hand, if $z_R \gg L$, diffraction coupling is relatively weak and the central portion of the beam may switch up while the wings of the beam remain on the lower branch (part-beam switching). This scenario neglects at least one important feature. Near the switching edge between the 'on' and 'off' portions of the medium, the diffraction length is characterized not by z_R, but more properly by $\pi d^2/\lambda$, where d is a length characteristic of the width of the transition region between the 'on' and 'off' regions. A sharp switching edge will thus diffract strongly into the 'off' region. This will both smooth the edge and tend to 'pull up' the unswitched region. A 'switching wave' may thus propagate outward (Rosanov & Semenov 1981) until that entire portion of the beam for which an upper branch exists is 'on', except for the transition region. In such a case, examination of the power output on a timescale long compared with the time needed for this process will see a substantial power step, roughly equal to the plane-wave intensity step times the area over which an upper branch exists. On lowering the input power, the 'on' region will shrink, until its area is too small to sustain itself against diffraction. The switch-down will thus involve a power step smaller than that on switch-up by a factor roughly equal to the ratio of these areas. If this is small, the power hysteresis characteristic will be rather triangular in shape, whereas that for the central intensity will be much squarer (Gibbs *et al.* 1982).

Diffraction derives from an optical amplitude gradient: the other transverse effects that we shall discuss derive from the refractive index gradient in the medium. First of these is (nonlinear) refraction; the induced refractive index causes the medium to act as a lens. If $n_2 > 0$, the lens is converging, and the beam may break up into filaments (Moloney, this symposium). If $n_2 < 0$, as in most semiconductors including InSb, the lens action enhances the diffractive spreading: indeed it is much more important than diffraction in many cases.

An important side-effect of the phase gradient is an induced curvature of the internal phase fronts, which clearly leads to loss of finesse, and hence of bistability if large enough. This will be particularly serious in higher orders, and for $n_2 < 0$, whereas for $n_2 > 0$ it can actually be helpful in compensating for the diffractive effects.

If the nonlinear index is due to a mobile excitation, e.g. photocarriers in semiconductors, then these will diffuse in the presence of phase (concentration) gradients. This is discussed in more detail below.

After that fairly extensive preamble, we now set down the field equations describing these phenomena, which must in general be solved numerically. These are, assuming steady-state conditions:

$$\{(\partial/\partial z) - \tfrac{1}{2}\mathrm{i}\mathscr{F}\nabla_\tau^2 + \tfrac{1}{2}\alpha L\}\, F(\boldsymbol{r}) = \mathrm{i}h(\boldsymbol{r})\, F(\boldsymbol{r}), \tag{4}$$

$$\{(-\partial/\partial z) - \tfrac{1}{2}\mathrm{i}\mathscr{F}\nabla_\tau^2 + \tfrac{1}{2}\alpha L\}\, B(\boldsymbol{r}) = \mathrm{i}h(\boldsymbol{r})\, B(\boldsymbol{r}), \tag{5}$$

$$(-l_\mathrm{d}^2\nabla_\tau^2 + 1)\, h(\boldsymbol{r}) = -4\mathscr{F}\,\mathrm{sgn}\,(n_2)\,(|F(\boldsymbol{r})|^2 + |B(\boldsymbol{r})|^2). \tag{6}$$

Here the transverse coordinates are scaled to w, ∇_τ^2 is the transverse Laplacian; the longitudinal coordinate is scaled to L, and $\mathscr{F}$ equals $L/2z_R$. The forward and backward field amplitudes, F and B, are scaled so that for example the forward power is

$$P_\mathrm{f}(z) = (2P_\mathrm{c}/\pi)\iint |F|^2\,\mathrm{d}x\,\mathrm{d}y,$$

$$P_\mathrm{c} = \lambda_0^2/2\pi n_0\,|n_2|.$$

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P_c is the 'critical power for self-focusing' in the case $n_2 > 0$. The third field $h(\boldsymbol{r})$ is the excitation density, scaled to be the nonlinear phase shift per unit propagation distance. It is assumed to diffuse transversely with a diffusion length l_d (longitudinal diffusion is less significant). In the limit $l_d \rightarrow 0$, (6) gives an explicit expression for $h(\boldsymbol{r})$ that may be inserted into (4) and (5). The resulting equations are valid only if $l_d \gtrsim \lambda$, since for $l_d \lesssim \lambda$ one must include population grating terms, which, in the limit $l_d \rightarrow 0$, have the effect of doubling the phase shift imposed by the counter-propagating fields on each other (termed nonlinear non-reciprocity by Kaplan & Meystre 1982). The boundary conditions are

$$B(r, 1) = R_b^{\frac{1}{2}} F(r, 1), \tag{7}$$

$$F(r, 0) = T_f^{\frac{1}{2}} I_{in} + R_f^{\frac{1}{2}} B(r, 0)\, e^{-2i\delta}, \tag{8}$$

essentially as for plane waves.

Only numerical solutions of this system of complex nonlinear partial differential equations have been obtained. Only recently (Moloney, this symposium) has a full two-dimensional problem been examined; most authors have either eliminated one cartesian coordinate or assumed cylindrical symmetry. Methods used have included mode expansions (Ballagh *et al.* 1981; Firth & Wright 1982), which operate in real space but necessarily truncate the mode series; and fast transform techniques (Rosanov & Semenov 1981; Moloney & Gibbs 1982; Moloney 1982 and this symposium) in which the nonlinearity is handled by reducing the medium to a set of slices, and the (linear) propagation between slices is treated by a fast transform algorithm (Siegman 1977).

We report here some new results obtained in the cylindrical geometry using he fast-Hankel transform (f.H.t.) technique for parameters identical to those used by Firth & Wright (1981) with mode expansions. For the chosen parameters, broadly representative of InSb experiments, the plane-wave model gives bistability in first order. Firth & Wright found large bistable loops for low mode numbers, but as more transverse modes were included, these loops narrowed rapidly. By extrapolation beyond the highest mode number used (six), the authors concluded that power hysteresis would probably vanish. Our new results (figure 2), from the use of four slices in the f.H.t. calculation, actually show power hysteresis, but extremely small. With regard

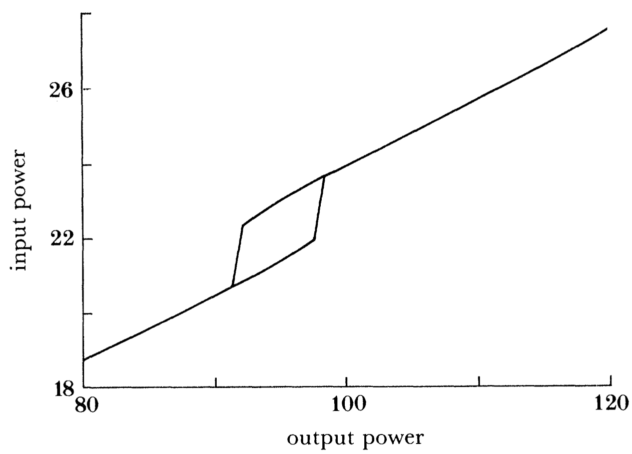

FIGURE 2. Output power as a function of input power, calculated by using the f.H.t. method. $\mathscr{F} = 1/128$, $Re^{-\alpha D} = 0.218$, $\delta = 0$.

to intensity profiles, Firth & Wright found smooth quasi-Gaussian profiles on the lower branch, switching to a peak-and-ring profile on the upper branch. The f.H.t. calculations (figure 3) broadly confirm this.

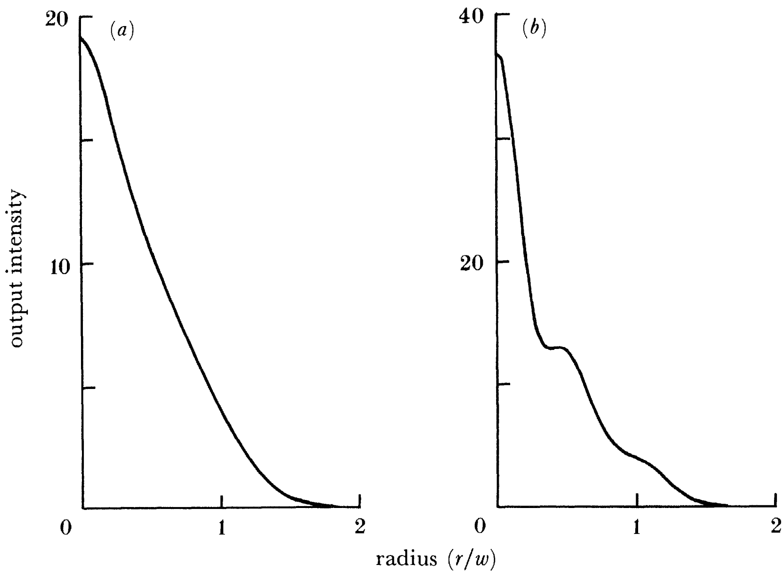

FIGURE 3. Near-field intensity profiles for (a) 'off' and (b) 'on' states. Parameters as in figure 2.

Firth & Wright postulated that this beam reshaping could occur, and show hysteresis, even in the absence of power hysteresis, a phenomenon that they termed 'spatial hysteresis'. Our new results do not show this, though ironically there are two reports (Dagenais & Winful (1984) in CdS, Walker (this symposium) in InSb) in which spatial hysteresis is observed with no apparent power hysteresis. In contrast to the opening discussion in this section, it would seem that the particular parameters used in our computations happen to lie in the transition region between whole-beam switching and part-beam switching, and are thus particularly sensitive to the algorithm and approximations used.

4. Diffusion

In this section we present some new analysis and results on transverse effects in optical bistability where diffusion (of the excitation) is the primary mechanism coupling neighbouring regions of the nonlinear medium. Whereas diffraction or nonlinear lensing, or both, will establish a correlation over regions of area *ca.* λL (see §3), diffusion will be effective over areas of order l_d^2 which, if larger than λL, may lead to whole-beam switching where a purely local theory would predict part-beam switching. The minimum pixel size and power for bistability will then also be determined by l_d^2 rather than λL.

Diffusion has previously been considered mainly with regard to grating effects (see above), but Rosanov (1981) has examined transverse diffusion, though primarily in regard to switching waves analogous to those in diffractive coupling.

There are a number of reasons for believing transverse diffusion to be significant in InSb, not least the fact that material parameters lead to an estimate of *ca.* 70 μm for l_d at 77 K

(H. A. MacKenzie, private communication); square hysteresis loops are observed despite fairly high Fresnel number, and multiple orders are obtained despite nonlinear lensing and finesse destruction. Diffusion also manifests itself as an increase in the central intensity needed to obtain a given nonlinear phase shift in a (single-pass) defocusing experiment, if the beam area becomes comparable with l_d^2. Preliminary measurements in InSb (F. P. Tooley, private communication) show such an effect for $w \approx 100$ μm. In fact a variational-Gaussian approximation to the carrier profile (6) in a single pass experiment gives a width larger than w by a factor $f^{-\frac{1}{2}}$:

$$f = \{(1+32\, l_d^2/w^2)^{\frac{1}{2}} - 1\}/(16\, l_d^2/w^2).$$

It will be seen that $f = 1$ for $l_d = 0$, but $f = \frac{1}{2}$ even for $l_d = \frac{1}{2}w$: the corresponding central phase shift is down by a factor $\frac{4}{9}$ in this case.

Turning now to bistability, we choose the simplest possible model, neglecting diffraction and considering only one cartesian transverse coordinate. We can then integrate the field equations (4, 5), and obtain a simple differential equation for the transverse phase profile:

$$-l_d^2\, \partial^2\phi/\partial x^2 + (\phi+\delta) = \theta\, e^{-x^2/w^2}/(1+F\sin^2\phi). \qquad (9)$$

Here θ is proportional to the input intensity, and can be seen to equal the plane-wave on-resonance nonlinear phase shift (equations (2, 3)). The required boundary conditions are that $\partial\phi/\partial x$ be zero on axis, and that $(\phi+\delta) \approx e^{-x/l_d}$ for $x \gg w$.

Figure 4 shows a hysteresis curve obtained by solving (9) for $F = \frac{1}{2}$ and $w = 2l_d$. Bistability is obtained in second order, and both the on-axis phase shift and the 'power' transmission show the squarish hysteresis loops characteristic of whole-beam switching, which confirms that a diffusion length of *ca.* 50 μm could explain the hysteresis behaviour in InSb.

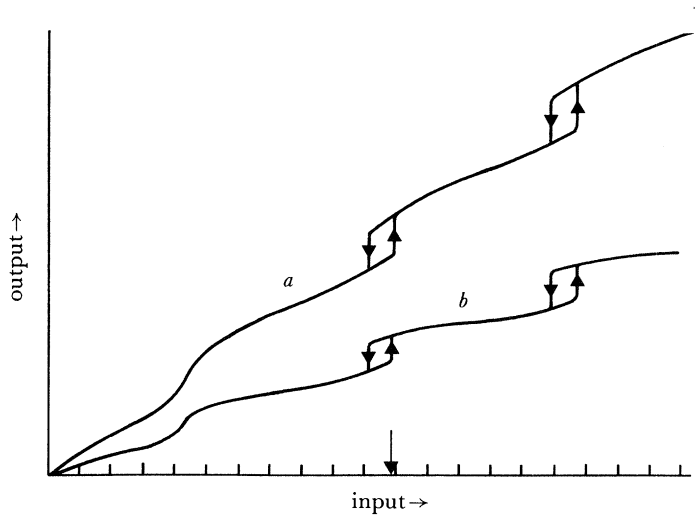

FIGURE 4. Dependence of (*a*) output 'power', and (*b*) on-axis carrier density, on input power: $l_d = \frac{1}{2}w$, $Re^{-\alpha D} = 0.1$, $\delta = 0$.

In computation of figure 4, all phase profiles were smooth. Figure 5 shows intensity profiles corresponding to the lower bistable branch of figure 4, with shoulders arising from the high transmission at $\phi = \pi$. Clearly the neglect of diffraction would be marginal for high-finesse resonators unless $l_d^2 \gg \lambda L$.

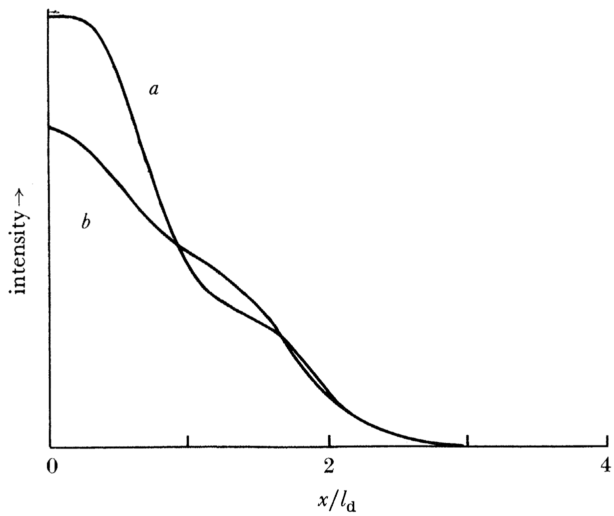

FIGURE 5. Intra-cavity intensity profiles for (*a*) 'on' and (*b*) 'off' states corresponding to the arrowed input in figure 4.

5. Conclusions

In this paper we have reviewed the salient features of macroscopic bistability theory. We have identified a number of advantages of reflection-mode operation over the conventional transmission mode, discussed the 'transverse effects' and presented some comparative data on diffraction coupling, and, finally, advanced the view that diffusion can explain certain anomalies in the InSb bistability experiments, and supported this with some new numerical results.

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